Math 530 – Applied and Computational Algebra

Course Description (Bulletin): Basics of computation with systems of polynomial equations, ideals in polynomial rings; solving systems of equations by Groebner bases; introduction to elimination theory; algebraic varieties in affine n-space; Zariski topology; dimension, degree, their computation and theoretical consequences. (3-0-3) Credit may not be granted for both MATH 431 and MATH 530.

Enrollment: Elective for AM and other majors.

Textbook(s): Cox, Little and O'Shea: Ideals, Varieties and Algorithms: An Introduction to Computational Algebraic Geometry and Commutative Algebra. ISBN 978-1-4419-2257-1 (3rd edition).

Alternate textbook resource: Cox, Little and O'Shea: Using Algebraic Geometry, Graduate Texts in Mathematics, Springer, ISBN 978-0-387-27105-7 (2nd edition).

Other required material: Use of computer algebra system, such as Macaulay2, Singular, CoCoA, or Sage. All are free/open source.

Prerequisites: MATH 332 or MATH 532

Objectives:

- 1. Students will achieve command of the essentials of computational algebraic geometry and commutative algebra.
- 2. Students will understand and apply the core definitions and theorems, generating examples as needed, and asking the next na(t)-6.exu-6.6(a) 8Tvarieties and existence proofs.
- 4. Students will achieve proficiency in written and oral communication of proofs and concepts of both pure and applied computational algebraic geometry.
- 5. Students will become familiar with the major viewpoints and goals of algebraic geometry: ideals, varieties, and algorithms.
- Students will understand the basic geometric notions of dimension and degree of algebraic sets, the Zariski topology, and its consequences on solving systems of polynomial equations.
- 7. Students will practice their knowledge of advanced abstract algebra to problems with exercises and applications, through the required use of a computer algebra system, as well as a class project which will consist of reading an extra chapter or a research paper on the topic from the course.

Lecture schedule: 3 50 minute (or 2 75 minute) lectures per week

Course Outline:

Торіс	Hours
What is applied algebra? Preliminaries: basic introduction to algebraic structures (fields, rings).	3
Polynomials and affine spaces. Affine varieties and their parametrizations. Dimension and degree.	6
Ideals in the polynomial ring.	3
Polynomials in one variable, and Introduction to algorithms/pseudocode.	3
Monomial orderings and division algorithm in many variables. Dickson's Lemma. The Hilbert basis theorem. The ascending chain condition.	