Math 578 – Computational Mathematics II

Course Description from Bulletin: Polynomial interpolation; numerical integration; numerical solution of ordinary differential equations by single and multi-step methods, Runge-Kutta, Predictor-Corrector; numerical solution of boundary value problems for ordinary differential equations by shooting methods, finite differences and spectral methods. Credit may not be granted for both MATH 578 and MATH 478. (3-0-3)

Enrollment: Elective for AM and other majors.

Textbook(s): A. Iserles, *A First Course in the Numerical Analysis of Differential Equations*, Cambridge University Press (1996), ISBN 0-521-55655-4 (paperback).
D. Kincaid and W. Cheney, *Numerical Analysis: Mathematics of Scientific Computing*, 3rd Ed, Brooks/Cole (2002), ISBN 0-534-38905-8.

Other required material: Matlab

Prerequisites: An undergraduate numerical course such as MATH 350, or consent of the instructor

Objectives:

1. Students will understand the basic numerical methods for solving initial value problems and their derivations.
2. Students will understand the concepts of order, stability, and convergence of a numerical method.
3. Students will understand the basic numerical methods for solving boundary value problems and their derivations.
4. Students will learn how to implement and use these numerical methods in Matlab (or another similar software package).
5. Students will improve their problem solving skills in computational mathematics.
6. Students will improve their presentation and writing skills.

Lecture schedule: 3 50 minutes (or 2 75 minutes) lectures per week

Course Outline: Hours

1. Mathematical background 10
    a. Lipschitz continuity
    b. Taylor polynomials and polynomial interpolation, splines
    c. Numerical integration methods
    d. Richardson Extrapolation
    e. Existence and uniqueness theorem for initial value problems
2. Nonlinear algebraic systems 3
    a. Fixed-point iteration
    b. Newton-Raphson iteration
3. Single step methods for differential equations 5
    a. Derivation of Euler and Taylor methods, trapezoidal rule, theta method

b. Order and convergence

4. Multistep methods for differential equations 7
   a. Derivation of Adams methods, general multistep methods, BDFs
   b. Order and convergence
   c. Dahlquist equivalence theorem
5. Runge-Kutta methods 3
   a. Derivation
   b. General form
6. Stability and Stiff equations 3
   a. Linear stability analysis
   b. Stiffness
   c. A-Stability
7. Error control 4
   a. Adaptive stepsize control
   b. Predictor-Corrector methods
   c. Embedded Runge-Kutta methods
8. Boundary value problems 7
   a. Shooting methods
   b. Finite differences
   c. FFT and spectral method

Assessment:

Homework	10-30%
Computer Programs/Project	10-20%
Quizzes/Tests	20-50%
Final Exam	30-50%

Syllabus prepared by: Greg Fasshauer and Xiaofan Li
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