

MATH 579 – Complexity of Numerical Problems

Course Description from Bulletin: This course is concerned with a branch of complexity theory. It studies the intrinsic complexity of numerical problems, that is, the minimum effort required for the approximate solution of a given problem up to a given error. Based on a precise theoretical foundation, lower bounds are established, i.e. bounds that hold for all algorithms. We also study the optimality of known algorithms, and describe ways to develop new algorithms if the known ones are not optimal. (3-0-3).

Enrollment: Elective for AM and CS graduate students

Textbook(s)/References: H. Niederreiter, *Random Number Generation and Quasi-Monte Carlo Methods*, CBMS-NSF Regional Conference Series in Applied Mathematics, SIAM, Philadelphia, 1992.
E. Novak, *Deterministic and stochastic error bounds in numerical analysis*, Springer-Verlag, Lectures Notes in Mathematics Vol. 1349, Berlin, 1988.
K. Ritter, *Average-Case Analysis of Numerical Problems*, Lecture Notes in Mathematics Vol. 1733, Springer-Verlag, Berlin, 2000.
I. H. Sloan and S. Joe, *Lattice Methods for Multiple Integration*, Oxford University Press, Oxford, 1994
V. N. Temlyakov, *Approximation of Periodic Functions*, Computational Mathematics and Analysis Series, Nova Science Publishers, Inc., Commack, NY, 1993.
J. F. Traub, G. W. Wasilkowski and H. Wozniakowski,

Course Outline:	Hours
1. General Theory	6
a. What is The Complexity of Numerical Problems?	
b. Basic Notions	
c. Complexity of the Search Problem	
2. Integration	12
a. Reproducing kernel Hilbert spaces and error bounds	
b. Lower bounds on the complexity	
c. Product rules versus quasi-Monte Carlo Rules	
3. Tractability	6
a. How error bounds depend on the dimension	
b. What problems and algorithms yield weak dependence on the dimension	
4. Randomized and Average Case Complexity	6
5. Complexity and Tractability for Approximation	6
6. Selected topics	6

Assessment:	Homework	25–30%
	Presentation	25–30%
	Final Exam	40–50%

Syllabus prepared by: Fred Hickernell and Greg Fasshauer

Date: 01/12/06